Lecture course *Statistics II*
Winter semester 2016/17
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Exercise sheet 2

Exercise 1. Given $\Theta = [a, b] \subset \mathbb{R}$ let $M_n : \Theta \to \mathbb{R}$ be random functions and $M : \Theta \to \mathbb{R}$ be a deterministic function. Show that $\sup_{\theta\in\Theta} |M_n(\theta) - M(\theta)| = o_{\mathbb{P}}(1)$, if the following conditions are satisfied:

(a) M is continuous;

(b) $\theta \mapsto M_n(\theta)$ is monotone increasing in θ for all $n \in \mathbb{N}$;

(c) $M_n(\theta) = M(\theta) + o_{\mathbb{P}}(1)$ for all $\theta \in \Theta$.

Remark: By considering $-M_n$ the second assumption could also be monotone decreasing.

(4 points)

Exercise 2. Let $Z_1, \dots, Z_n$ be i.i.d. r.v.'s from a distribution with continuous density $f : \mathbb{R} \to \mathbb{R}$ with respect to the Lebesgue measure and distribution function F. Let θ_o be the median of F and assume that for all $\theta_1 < \theta_0 < \theta_2$ we have $F(\theta_1) < \frac{1}{2} < F(\theta_2)$. Show that the minimizer $\widehat{\theta}_n$ of $M_n(\theta) := \frac{1}{n}\sum_{i=1}^n |Z_i - \theta| - |Z_i|$ is a consistent estimator of the median $\theta_o \in \Theta = \mathbb{R}$ by proving the two conditions of Theorem §2.2.1.

(4 points)

Exercise 3. Let the assumptions of Theorem §2.3.1 be satisfied and let $\check{\theta}$ be another estimator of θ such that $\left\|\check{\theta} - \theta\right\| = O_{\mathbb{P}}(n^{-1/2})$. Consider the following up date of this estimator:

$$\tilde{\theta} = \check{\theta} - (\dot{\Psi}_n(\check{\theta}))^{-1}\Psi_n(\check{\theta}).$$

Show that $\widetilde{\theta} = \widehat{\theta} + o_{\mathbb{P}}(n^{-1/2})$. Conclude from the last equation that $\sqrt{n}(\widetilde{\theta} - \theta_o)$ has the same asymptotic normal limit as $\sqrt{n}(\widehat{\theta} - \theta_o)$.

(4 points)

Exercise 4. Consider a sequence of i.i.d. $\mathfrak{N}(\mu_o, \sigma_o^2)$ r.v.'s where $\theta_o := (\mu_o, \sigma_o)$ is an unknown parameter with $\mu_o \neq 0$. Construct the Wald-Test for the hypothesis $\sigma_o/\mu_o = 1$.

(4 points)

Handing in on **Friday, November 11, 2016** in **fixed groups of two**.