Lecture course *Statistics II*
Winter semester 2016/17
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Exercise sheet 1

Exercise 1. Prove the following statements about stochastic Landau symbols:

(a) $o_{\mathbb{P}}(1) + o_{\mathbb{P}}(1) = o_{\mathbb{P}}(1)$

(b) $(1 + o_{\mathbb{P}}(1))^{-1} = O_{\mathbb{P}}(1)$

(c) $o_{\mathbb{P}}(1) = O_{\mathbb{P}}(1)$

(d) $O_{\mathbb{P}}(1) + O_{\mathbb{P}}(1) = O_{\mathbb{P}}(1)$

(e) $O_{\mathbb{P}}(1) o_{\mathbb{P}}(1) = o_{\mathbb{P}}(1)$

(f) $o_{\mathbb{P}}(O_{\mathbb{P}}(1)) = o_{\mathbb{P}}(1)$

Hints: The equations mean that sequences of r.v.'s in $o_{\mathbb{P}}(1)$ or $O_{\mathbb{P}}(1)$ combined according to the left hand side of the equations, yield a sequence with the property on the right hand side. For instance, (e) means that if $X_n = O_{\mathbb{P}}(1)$ and $Y_n = o_{\mathbb{P}}(1)$ then $X_n \cdot Y_n = o_{\mathbb{P}}(1)$. For (b) and (e) you may consider real valued r.v.'s only. (4 points)

Exercise 2. Let $k \in \mathbb{N}$ and $g : \mathbb{R}^k \to \mathbb{R}$ with $g(0) = 0$. Assume $(X_n)_{n\in\mathbb{N}}$ to be a sequence of $\mathbb{R}^k$-valued r.v.'s with $X_n = o_{\mathbb{P}}(1)$. Show the following statements:

(a) If $g(x) = o(\|x\|^r)$ as $x \to 0$, then $g(X_n) = o_{\mathbb{P}}(\|X_n\|^r)$.

(b) If $g(x) = O(\|x\|^r)$ as $x \to 0$ then $g(X_n) = O_{\mathbb{P}}(\|X_n\|^r)$.

Hints: We say that $g(x) = o(\|x\|^r)$ as $x \to 0$ if $\frac{g(x)}{\|x\|^r} \to 0$ as $x \to 0$, and that $g(x) = O(\|x\|^r)$ as $x \to 0$, if there exists $\varepsilon > 0$ with $\sup_{\|x\|<\varepsilon} \frac{g(x)}{\|x\|^r} < \infty$. (4 points)

Exercise 3. Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of r.v.'s and $(A_n)_{n\in\mathbb{N}}$ a sequence of events. We say that "$X_n = o_{\mathbb{P}}(1)$ on A_n", if for all $\varepsilon > 0$, $\mathbb{P}(|X_n| > \varepsilon,\, A_n) \to 0$ as $n \to \infty$. Show that $X_n = o_{\mathbb{P}}(1)$ if $\mathbb{P}(A_n) \to 1$ as $n \to \infty$. (4 points)

Exercise 4. *(Lemma §5.2.3)* Assume that (i) Θ is compact, (ii) $M : \Theta \to \mathbb{R}$ is continuous and (iii) M has a unique maximum θ_0, i.e., there exists $\theta_0 \in \Theta$ such that for all $\theta \in \Theta$: $M(\theta) < M(\theta_0)$. Show that $\sup_{\theta\in\Theta:\, d(\theta,\theta_0)>\varepsilon} M(\theta) < M(\theta_0)$, for all $\varepsilon > 0$. (4 points)

Handing in on **Friday, November 04, 2016** in **fixed groups of two**.