Functional linear regression with functional response*

David Benatia
Université de Montréal

Marine Carrasco
Université de Montréal

Jean-Pierre Florens
Toulouse School of Economics

June 2016

Abstract

In this paper, we develop new estimation results for functional regressions where both the regressor $Z(t)$ and the response $Y(t)$ are functions of an index such as the time or a spatial location. Both $Z(t)$ and $Y(t)$ are assumed to belong to Hilbert spaces. The model can be thought as a generalization of the multivariate regression where the regression coefficient is now an unknown operator Π. An interesting feature of our model is that $Y(t)$ may depend not only on concurrent $Z(t)$ but also on $Z(s)$ for s in a neighborhood of t.

We propose to estimate the operator Π by Tikhonov regularization, which amounts to apply a penalty on the L^2 norm of Π. We derive the rate of convergence of the mean-square error, the asymptotic distribution of the estimator, and develop tests on Π. Often, the full trajectories are not observed but only a discretized version is available. We address this issue in the scenario where the data become more and more frequent (in-fill asymptotics). We also consider the case where Z is endogenous and instrumental variables are used to estimate Π.

Key Words: Functional regression, instrumental variables, linear operator,Tikhonov regularization

*The authors thank the participants of the 6th French Econometrics Conférence and especially their discussant Jan Johannes for helpful comments. Stéphane Bonhomme is also thanked for helpful comments. Carrasco thanks NSERC for financial support.